UNIT-1

INTRODUCTION

System:

An arrangement (or) combination of different physical components that are connected (or) related together to form an entire unit to achieve a certain objective is called a system

Control:

The meaning of control is to regulate, direct (or) command a system so that a desired objective is obtained.

Control System:

A control system is an arrangement of physical components connected or related in such a manner as to command, direct or regulate itself or another system to obtain a certain objective.

Input :

The excitation or stimulus applied to a Control system from an external energy source is usually known as input.

output:

The actual Response that is obtained from a Control system due to the application of input is called output.

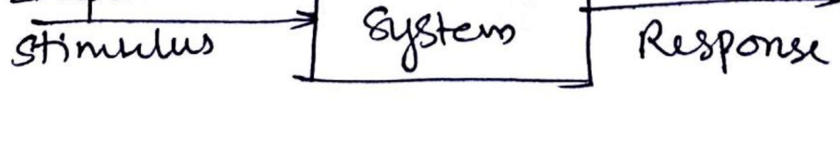
plant (or) process:

It is defined as portion of a system which is to be Controlled or regulated. It is also called process.

Controller

It is an element with in the system itself or external to the system and it controls the plant or the process.

In its simplest form, a Control system is shown below



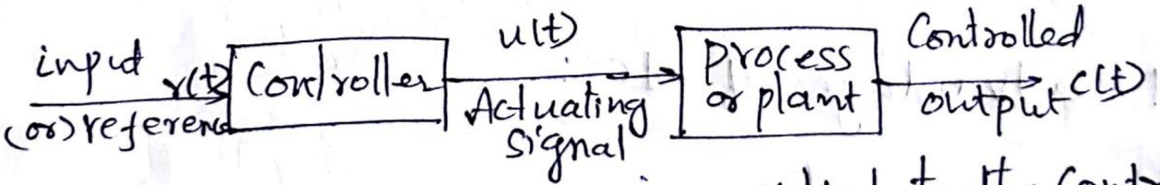
Control system Configurations:

There are two major configurations of control systems.

1. open loop System
2. Closed loop System.

① open-loop System:

- An open-loop control system is one in which the output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system.
- The open-loop system is also called the non-feedback System.
- The below figure shows the open-loop System



Reference input $r(t)$ is applied to the controller which generates the actuating signal $u(t)$ required to control the process which is to be controlled. process gives the desired controlled output $c(t)$.

- For a given input the System produces a Certain output. If there are any disturbances, the output changes and there is no adjustment of the input to bring back the output to the original value.

Some Examples of open-loop Systems are :

1. Bread toaster
2. Automatic washing machine.
3. Traffic signals etc.,

Advantages of OLCS:

1. simple in construction.
2. These Systems are Economical
3. No stability problem

Disadvantages of OLCS:

1. Less accurate
2. Recalibration of the Controller is required.

Closed-loop system:

- A System in which the controlling action or input is somehow dependent on the output or changes in output is called closed-loop System

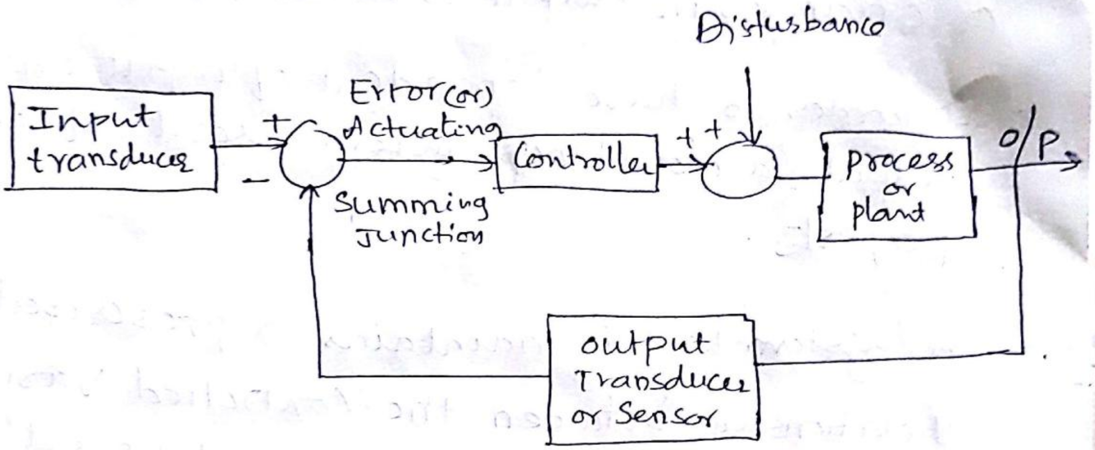
- Inorder to have dependent of input on the output a closed loop system uses the feedback property.

- A system which maintains a prescribed relationship between the controlled variable and the reference input, and uses the difference between them as a signal to activate the control is known as feedback control system. In such a system, output or part of the output is fedback to the input for comparision with the reference input and an actuating signal is generated.

- closed loop control systems are called feed back control systems

- The disadvantages of open-loop systems, namely sensitivity to disturbances and inability to correct for these disturbances, may be over-come in closed-loop Systems.

The below figure shows the closed-loop system:
The input to the entire system is called reference input or command input, r(t).



- The input transducer converts the form of the one input to the form used by the controller.

- In the above figure the output signal is substracted from the input signal

- If there is any difference between the o/p and reference input, the system drives the plant via actuating signal to make correction. If there is no difference, the system does not drive the plant, since the system response is already desired response.

- Thus closed loop system monitor the o/p and compare it to the i/p. If an error is detected, the system corrects the o/p and hence corrects the effect of disturbance.

Comparision between open loop and closed loop control systems:

	Openloop System	Closed loop System
1.	The Feedback Element is absent	The Feedback element is always present
2	It is stable one	It may become unstable
3	It easy to construct	complicated Construction
4	It is an Economical System	It is costly
5	Having Small bandwidth	Having smaller band-width
6	It is inaccurate.	It is accurate
7	Less maintenance	More Maintenance
8	It is Unreliable	It is reliable
9	Ex: Hand drier, Washing Machine	Ex: Servo Voltage Stabilizer.

Feedback characteristics:

1 Sensitivity Analysis:

A control system is said to be highly sensitive if its control objective (or) response is effected by any internal (or) external disturbances.

Sensitivity Function:

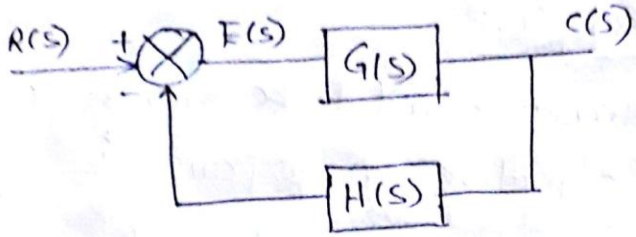
Let α = A variable that changes its value

β = A parameter that changes the value of α.

$$S_{\beta}^{\alpha} = \frac{\%\text{ change in } \alpha}{\%\text{ change in } \beta} = \frac{\frac{\partial \alpha}{\alpha}}{\frac{\partial \beta}{\beta}}$$

$$\boxed{\left| S_{\beta}^{\alpha} \right| = \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial \beta}}$$

Sensitivity analysis for closed loop control system:



case(i): Let α = closed loop transfer function = M(S)

β = Disturbances in forward path elements ie G(s)

$$S^{M(s)}_{G(s)} = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

Since $M(s) = \dfrac{G(s)}{1+G(s)H(s)}$

$$\frac{G(s)}{M(s)} = 1+G(s)H(s)$$

and
$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

$$= \frac{(1+G(s)H(s)) - G(s)H(s)}{[1+G(s)H(s)]^2} = \frac{1}{[1+G(s)H(s)]^2}$$

$$\therefore S^{M(s)}_{G(s)} = (1+G(s)H(s))\frac{1}{[1+G(s)H(s)]^2}$$

$$\boxed{S^{M(s)}_{G(s)} = \frac{1}{1+G(s)H(s)}}$$

where $1+G(s)H(s)$ is called noise reduction factor

Case VI: Let α = closed loop transfer function = $M(s)$

β = Disturbances in feedback path elements i.e, $H(s)$

$$S_{H(s)}^{M(s)} = \frac{H(s)}{M(s)} \frac{\partial M(s)}{\partial H(s)}$$

since $M(s) = \dfrac{G(s)}{1+G(s)H(s)}$

$$\frac{M(s)}{H(s)} = \frac{G(s)}{H(s)[1+G(s)H(s)]}$$

$$\frac{H(s)}{M(s)} = \frac{H(s)[1+G(s)H(s)]}{G(s)}$$

$$\frac{\partial M(s)}{\partial H(s)} = \frac{\partial}{\partial H(s)}\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

$$= \frac{(1+G(s)H(s)) * 0 - G(s) \cdot G(s)}{[1+G(s)H(s)]^2}$$

$$= \frac{-(G(s))^2}{(1+G(s)H(s))^2}$$

$$S_{H(s)}^{M(s)} = \frac{H(s)[1+G(s)H(s)]}{G(s)} * \frac{-[G(s)]^2}{[1+G(s)H(s)]^2}$$

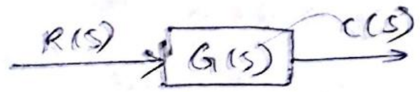
$$\boxed{S_{H(s)}^{M(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}}$$

Note: A closed loop (or) feedback control system is more sensitive to disturbances in feedback elements ie $H(s)$ than forward path elements ie $G(s)$.

Sensitivity of openloop system:

Let α = openloop transfer function = $M(s)$

$\beta = G(s)$



$$S^{M(s)}_{G(s)} = \frac{G(s)}{M(s)} \frac{\partial M(s)}{\partial G(s)}$$

Since $M(s) = G(s)$

$$\frac{\partial M(s)}{\partial G(s)} = 1$$

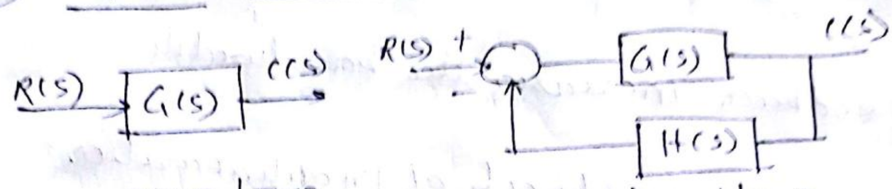
$$S^{M(s)}_{G(s)} = \frac{G(s)}{G(s)} \cdot 1 = 1$$

$$\boxed{S^{M(s)}_{G(s)} = 1}$$

$\therefore$ openloop systems are more sensitive to parameter variations.

Effects of feedback:

(1) Effect of feedback on overall gain:



open-loop closed loop

Consider an open-loop System with overall gain $G(s)$ as shown above.

If the feedback with transfer function $H(s)$ is introduced in such a system, the overall gain with -ve feedback becomes $\frac{G(s)}{1+G(s)H(s)}$

Therefore, for a -ve feedback the gain $G(s)$ is reduced by a factor $\frac{1}{1+G(s)H(s)}$

2. Effect of feedback on stability:

The Feedback can improve stability or be harmful to stability if its not properly applied.

3 Effect of feedback on sensitivity:

In general, a good control system should be very insensitive to parameter variations but sensitive to input commands. Feedback can increase or decrease the sensitivity of the system

4. Feedback reduces the effects of noise and disturbance on the system performance.

5. Feedback increases the bandwidth.

6. Reduces the effects of nonlinearities and distortion.

7. Stabilizes an unstable system.

8. Increases the accuracy by reducing the steady-state error.

Mathematical Model of physical Systems: ②

- For the analysis and design of Control Systems, we formulate a mathematical description of the System. The process of obtaining the desired mathematical description of the system is known as modelling.

- A mathematical model is not unique to a given system. A System may be represented in many different ways and, therefore, may have many mathematical models depending on requirement.

Modelling of Mechanical Systems:

- The motion of mechanical systems may be translational ~~systems~~, rotational or Combination of both.

- The motion of translation is defined as a motion that takes place along a straight line. The rotational motion of a body is defined as a motion about a fixed axis.

TRANSLATIONAL MOTION:

In translational motion, the systems are characterized by linear displacement, linear velocity and linear acceleration.

- The Equations of motion are formulated by Newton's laws of motion.

- Newton's law of motion states, that the algebraic sum of the forces acting on a rigid body in a given direction is equal to the product of the mass of the body and its acceleration in the same direction. This law can be expressed as

$$\sum Forces = Ma = M\frac{du}{dt} = M\frac{d^2x}{dt^2}$$

where M = mass of the body

a = acceleration of the body in the direction considered.

u = linear velocity of the body

x = linear displacement of the body

Network elements of a mechanical translational Systems:

There are three fundamental passive elements or System parameters which characterize a linear mechanical translational system. These are

(i) Mass

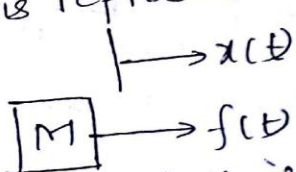
(ii) Spring

(iii) Friction

Mass (M):

In the physical model of a mass, it is assumed that its mass is concentrated at its centre. The function of Mass in Linear motion is to store kinetic energy.

Symbolically, mass is represented by a block shown below

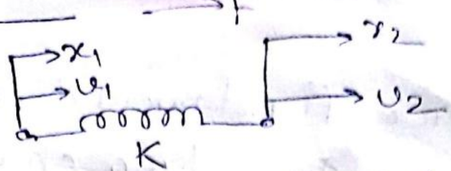


Suppose that a force $f(t)$ is applied to mass the applied force produces a displacement $x(t)$ in the direction of the applied force. The relation between applied force $f(t)$ & m is given by

$$f = ma = M\frac{du}{dt} = M\frac{d^2x}{dt^2}$$

$$\boxed{f = M\frac{d^2x}{dt^2}}$$

Spring Element:



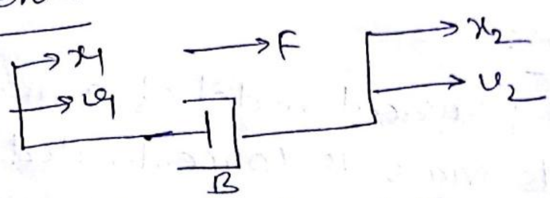
$$F = K(x_1 - x_2) = Kx$$

where $x_1 - x_2 = x$

$$F = K\int(v_1 - v_2) = K\int v$$

$$V = v_1 - v_2$$

Damper Element:



$$F = B(v_1 - v_2) = Bv$$

$$F = B\frac{d}{dt}(x_1 - x_2)$$

$$= B\dot{x} \quad (\text{where } x = x_1 - x_2)$$

Rotational Systems:

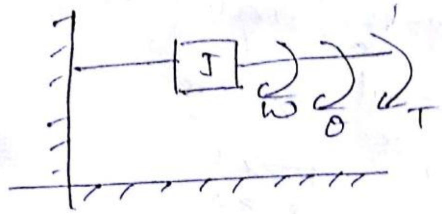
Input = torque (T)

output = Angular displacement θ

(or) Angular velocity (ω)

The 3 ideal Elements are

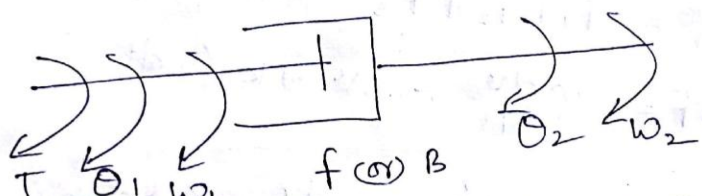
(1) Inertia Constant:



$$T = J\frac{d\omega}{dt}$$

$$T = J\frac{d^2\theta}{dt^2}$$

(2) Torsional Damper Element (Friction):



$$T = B(\omega_1-\omega_2) = B\omega \quad (\omega=\omega_1-\omega_2)$$

$$T = B\frac{d}{dt}(\theta_1-\theta_2) = B\frac{d\theta}{dt} \quad (\theta=\theta_1-\theta_2)$$

(3) Torsional Spring Element (Stiffness)

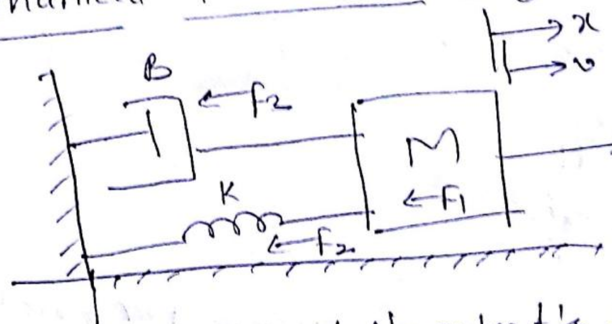
$$T = K\int \omega\, dt$$

$$T = K\theta.$$

Analogous Systems:

The electrical Equivalent of mechanical elements are known as analogous systems

① Mechanical Translational System:



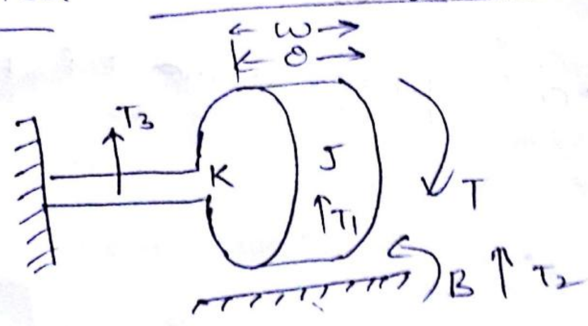
According to D' Alembert's law

$$F = F_1 + F_2 + F_3$$

$$F = M\frac{dv}{dt} + Bv + K\int v\,dt$$

$$\boxed{F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx} \rightarrow ①$$

② Mechanical Rotational System:

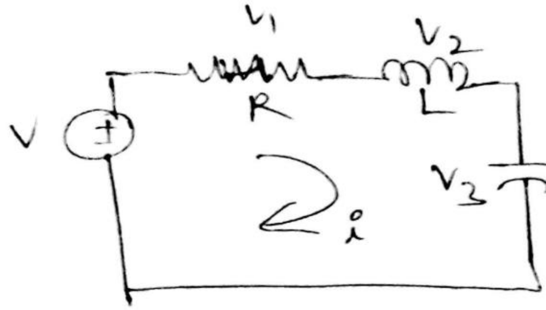


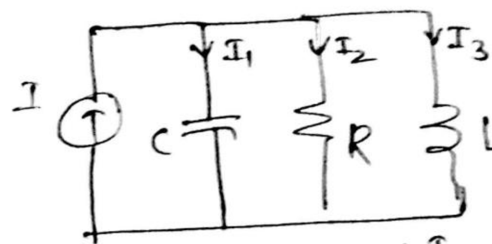
$$T = T_1 + T_2 + T_3$$

$$T = J\frac{d\omega}{dt} + B\omega + K\int \omega\,dt$$

$$\boxed{T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta} \rightarrow ②$$

) Electrical System :





$I = I_1 + I_2 + I_3$

$V = V_1 + V_2 + V_3$

$= iR + L\frac{di}{dt} + \frac{1}{c}\int i dt$

$i = \frac{dq}{dt}$, q = charge

$$\boxed{V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c}} \to (3)$$

$I = c\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v dt$

$\left(v = \frac{d\phi}{dt}, \phi = \text{flux}\right)$

$$\boxed{I = c\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L}} \to (4)$$

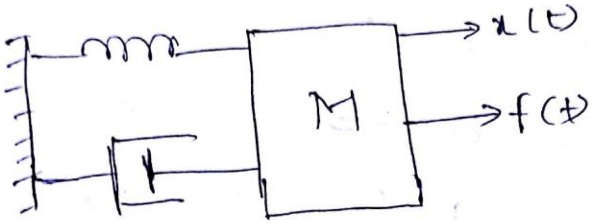
Compare equations (1) to (4)

(1) F - T - V - analogy
(2) F - T - I - analogy

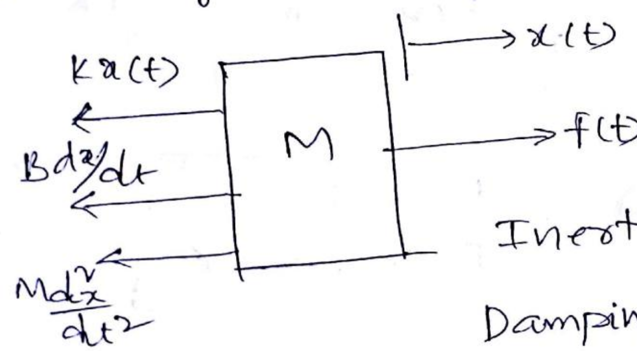
F	T	V	I
M	J	L	C
B	B	R	$\frac{1}{R}$
K	K	$\frac{1}{C}$	$\frac{1}{L}$

Problems:

① Obtain the differential equations governing the mechanical system shown below and find the transfer function $\frac{X(s)}{F(s)}$ for the system



Sol. Free body diagram



Inertia force, $f_m(t) = M\frac{d\tilde{x}}{dt}$

Damping force, $f_B(t) = B\frac{dx}{dt}$

Spring force, $f_k(t) = Kx(t)$

The inertia force, damping force and spring force and spring force impede the motion and act opposite to the direction of motion.

By Newton's law the algebraic sum of all the forces must be equal to zero.

$$\therefore \; f(t) = f_m(t) + f_B(t) + f_K(t)$$

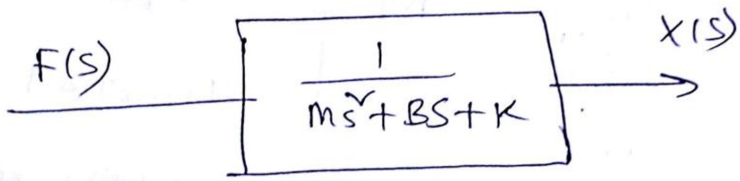
$$f(t) = M\frac{d^2x}{dt} + B\frac{dx}{dt} + Kx$$

Assuming zero initial conditions take the LT both the sides

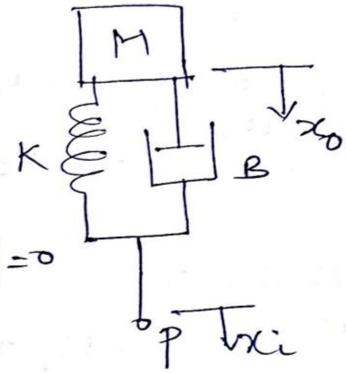
$$Ms^2X(s) + BsX(s) + KX(s) = F(s)$$

$$(Ms^2 + Bs + K)X(s) = F(s)$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}}$$



② Determine the transferfunction $\frac{X_o(s)}{X_i(s)}$ for the system shown below, assuming that the motion x_i at point P is the input to the system and and the motion x_o of the body is the output.



Sol:

$$M\frac{d^2x_o}{dt^2} + B\left[\frac{dx_o}{dt} - \frac{dx_i}{dt}\right] + K[x_o - x_i] = 0$$

$$M\frac{d^2x_o}{dt^2} + B\frac{dx_o}{dt} + Kx_o = B\frac{dx_i}{dt} + Kx_i$$

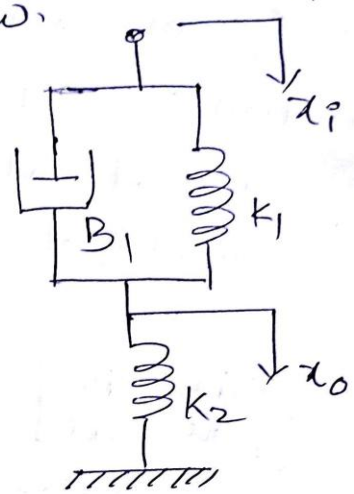
After taking LT

$$MS^2X_o(s) + BSX_o(s) + KX_o(s) = BX_i(s) + KX_i(s)$$

$$(MS^2 + BS + K)X_o(s) = (BS + K)X_i(s)$$

$$\boxed{\frac{X_o(s)}{X_i(s)} = \frac{BS+K}{MS^2+BS+K}}$$

(3) obtain the transfer function $\frac{X_o(s)}{X_i(s)}$ for the system shown below.



Sol: The equation of motion for the system is

$$B_1\left(\frac{dx_i}{dt}-\frac{dx_o}{dt}\right)+K_1(x_i-x_o) = K_2 x_o$$

$$B_1\frac{dx_i}{dt}+K_1x_i = B_1\frac{dx_o}{dt}+(K_1+K_2)x_o$$

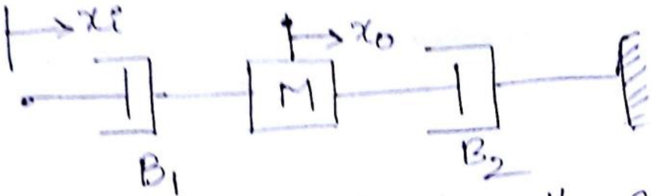
After taking LT

$$B_1 s X_i(s) + K_1 X_i(s) = B_1 s X_o(s) + (K_1+K_2)X_o(s)$$

$$(B_1 s + K_1) X_i(s) = (B_1 s + K_1 + K_2) X_o(s)$$

$$\boxed{\frac{X_o(s)}{X_i(s)} = \frac{B_1 s + K_1}{B_1 s + K_1 + K_2}}$$

(4) Determine the transfer function $\frac{X_0(S)}{X_i(S)}$



Sol. The equation of motion for the system is

$$B_1\left(\frac{dx_i}{dt}-\frac{dx_0}{dt}\right) = M\frac{d^2x_0}{dt^2}+B_2\frac{dx_0}{dt}$$

$$B_1\frac{dx_1}{dt} = M\frac{d^2x_0}{dt^2}+(B_1+B_2)\frac{dx_0}{dt}$$

After taking LT

$$B_1 S X_i(S) = MS^2X_0(S) + (B_1+B_2)SX_0(S)$$

$$B_1 S X_i(s) = \left[MS^2+(B_1+B_2)S\right]X_0(S)$$

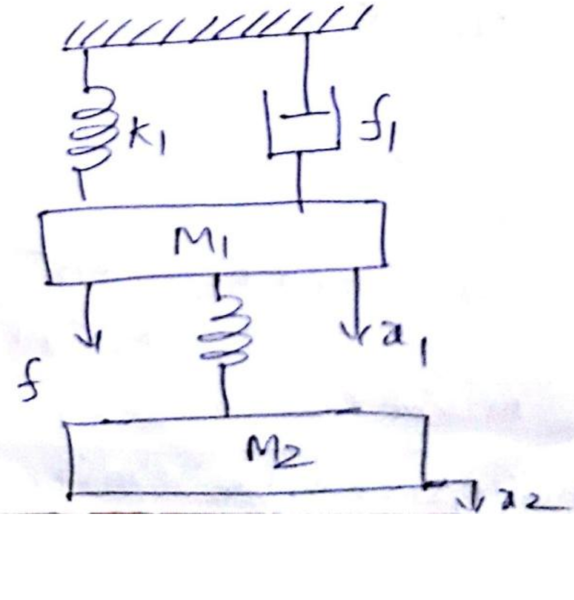
$$\frac{X_0(s)}{X_i(s)} = \frac{B_1 S}{MS^2+(B_1+B_2)S}$$

$$\boxed{\frac{X_0(s)}{X_i(s)} = \frac{B_1}{MS+(B_1+B_2)}}$$

Nodal Method for finding Differential equation of complex Mechanical System:

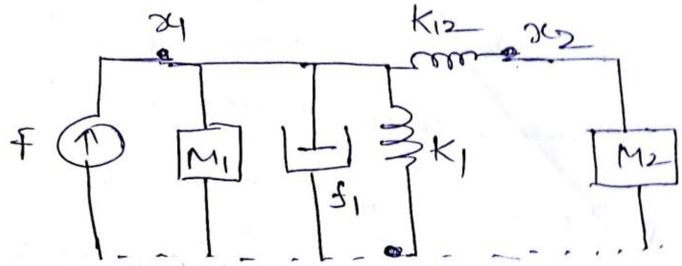
1. Number of nodes = Number of displacements
2. Take additional node which is reference node.
3. Connect the mass and inertial mass elements always between the principle node and reference node.
4. Connect the spring and damping elements either between the principle nodes or between principle nodes and reference depending on their position
5. Obtain the nodal diagram and write the describing equations at each node.

Ex: obtain t/F $\frac{X_1(s)}{F(s)}$ for the following system.



Nodal Diagram:

(Mechanical Equivalent diagram)



At Node x_1:

$$f = M_1 \frac{d^2 x_1}{dt^2} + f_1 \frac{dx_1}{dt} + K_1 x_1 + K_{12}(x_1 - x_2)$$

At Node x_2:

$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_{12}(x_2 - x_1)$$

Transfer Function:

$$\frac{X_1(s)}{F(s)}$$

$$F(s) = \left[M_1 s^2 + f_1 s + K_1 + K_{12}\right] X_1(s) - K_{12} X_2(s)$$

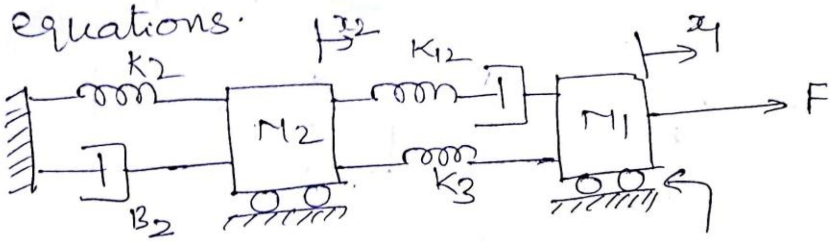
$$0 = (M_2 s^2 + K_{12}) X_2(s) - K_{12} X_1(s)$$

$$X_2(s) = \left(\frac{K_{12}}{M_2 s^2 + K_{12}}\right) X_1(s)$$

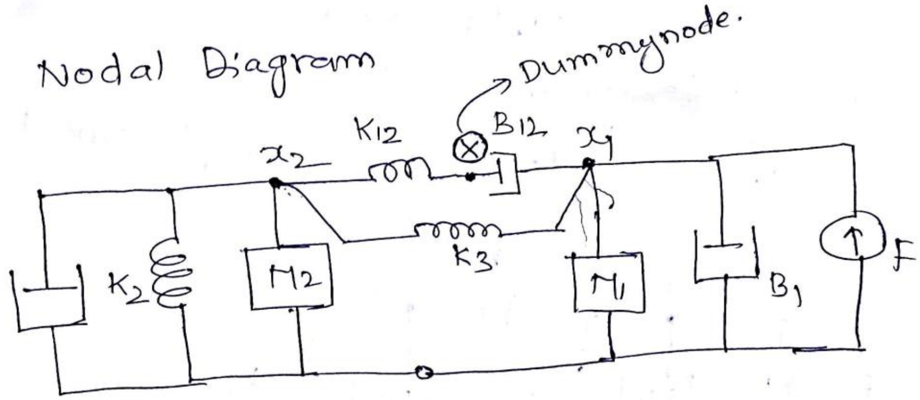
$$F(s) = \left\{M_1 s^2 + f_1 s + K_1 + K_{12} - \frac{K_{12}^2}{M_2 s^2 + K_{12}}\right\} X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + K_{12}}{(M_1 s^2 + f_1 s + K_1 + K_{12})(M_2 s^2 + K_{12}) - K_{12}^2}$$

→ For the following system write the differential equations.



Sol: Nodal Diagram



At Node ①:

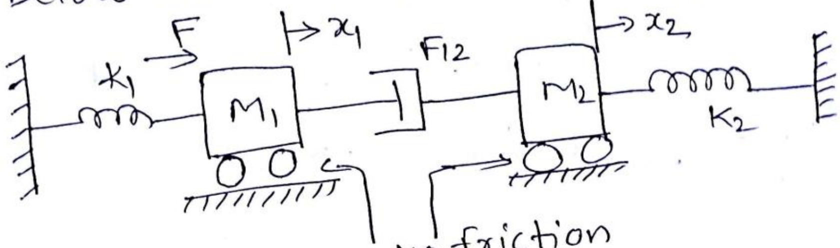
Apply newton's second law

$$F = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_3 (x_1 - x_2) + B_{12}(x_1 - x)$$

At Node ②

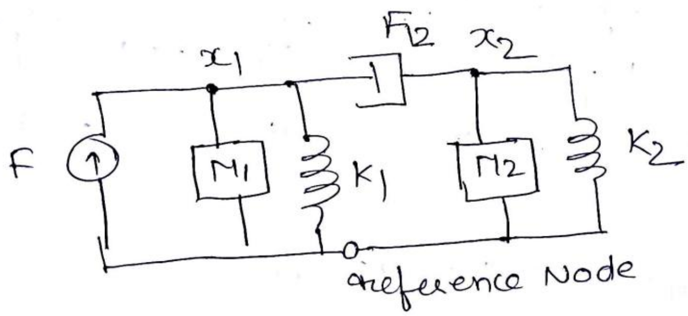
$$M_2 \frac{d^2 x}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + K_{12}(x_2 - x) = 0$$

→ For the mechanical System of figure shown below write the differential equations.



Sol: Nodal Diagram

No. of Nodes = No. of displacements



At Node 'x_1'

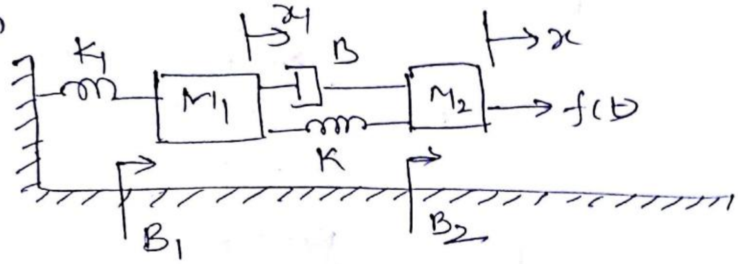
Apply newton's second law

$$F = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + F_{12} \frac{d}{dt}(x - x_1)$$

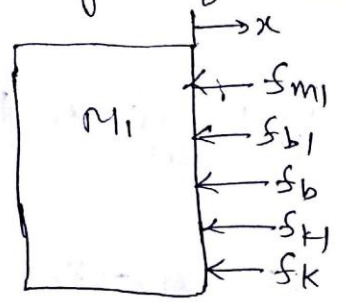
At Node 'x_2'

$$M_2 \frac{d^2 x_2}{dt^2} + F_{12} \frac{d}{dt}(x_2 - x_1) + K_2 x_2 = 0$$

→ Determine the transfer function for the following system



Sol: Free body diagram for mass M_1



By newton's second

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d(x_1-x)}{dt} + K_1 x_1 + K(x_1-x) = 0$$

Apply LT

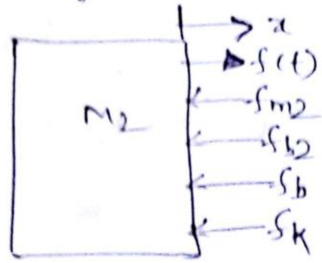
$$M_1 s^2 X_1(s) + B_1 s X_1(s) + Bs[X_1(s) - X(s)] + K_1 X_1(s) + K[X_1(s) - X(s)] = 0$$

$$X_1(s)[M_1 s^2 + (B_1+B)s + (K_1+K)] - X(s)[Bs+K] = 0$$

$$X_1(s) = X(s) \frac{Bs+K}{M_1 s^2 + (B_1+B)s + (K_1+K)} \longrightarrow (1)$$

owing

Free body diagram for M_2



By newton's second law

$$M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1) = f(t)$$

Apply LT

$$M_2 s^2 X(s) + B_2 s X(s) + Bs[X(s) - X_1(s)] + K[X(s) - X_1(s)] = F(s)$$

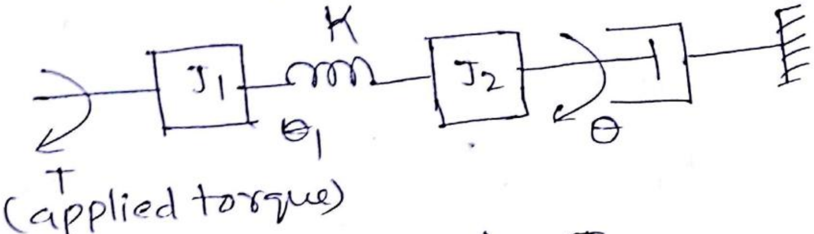
$$X(s)[M_2 s^2 + (B_2 + B)s + K] - X_1(s)[Bs + K] = F(s) \rightarrow ②$$

Substituting for $X_1(s)$ from equation ① in equation ②

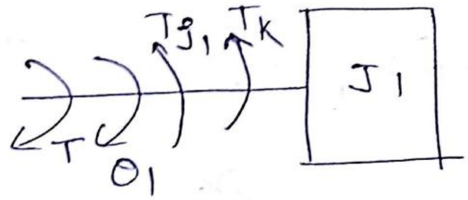
$$X(s)[M_2 s^2 + (B_2 + B)s + K] - X(s)\frac{(Bs+K)^2}{M_1 s^2 + (B_1 + B)s + (K_1 + K)} = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (K_1 + K)}{[M_1 s^2 + (B_1 + B)s + (K_1 + K)][M_2 s^2 + (B_2 + B)s + K] - (Bs+K)^2}$$

→ obtain the transfer function of the following rotation system shown below.



Sol: Free body diagram for J_1



By newtons second law

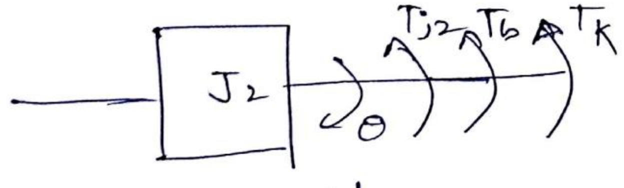
$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

Apply LT

$$J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta(s) = T(s)$$

$$(J_1 s^2 + K)\theta_1(s) - K\theta(s) = T(s) \rightarrow (1)$$

Free body diagram for J_2



By newton's second law

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

After applying LT

$$J_2 s^2 \theta(s) + Bs\,\theta(s) + K\theta(s) - K\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + K)\theta(s) - K\theta_1(s) = 0$$

$$\theta_1(s) = \frac{J_2 s^2 + Bs + K}{K}\,\theta(s)$$

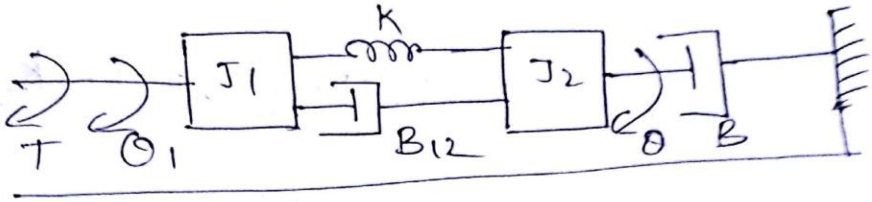
Substituting $\theta_1(s)$ in equation ①

$$(J_1 s^2 + K)\frac{(J_2 s^2 + Bs + K)}{K}\,\theta(s) - K\theta(s) = T(s)$$

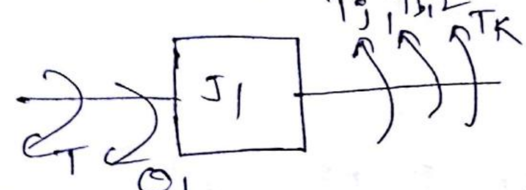
$$\left[\frac{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}{K}\right]\theta(s) = T(s)$$

$$\boxed{\frac{\theta(s)}{T(s)} = \frac{K}{(J_1 s^2 + K)(J_2 s^2 + Bs + K) - K^2}}$$

→ Determine the transfer function for the following rotational system.



Sol: Free body diagram for J_1



By newton's second law

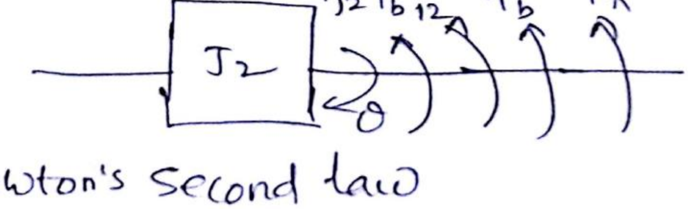
$$J_1\frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1-\theta) + K(\theta_1-\theta) = T$$

Apply LT

$$J_1 s^2\theta_1(s) + B_{12}[\theta_1(s)-\theta(s)] + K[\theta_1(s)-\theta(s)] = T(s)$$

$$\theta_1(s)\left[J_1 s^2 + sB_{12} + K\right] - \theta(s)\left[sB_{12} + K\right] = T(s) \quad \rightarrow (1)$$

Free body diagram for J_2



By newton's second law

$$J_2\frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}(\theta-\theta_1) + B\frac{d\theta}{dt} + K(\theta-\theta_1) = 0$$

$$J_2 s^2 \theta(s) - B_{12} s \theta_1(s) + s\theta(s)[B_{12}+B] + K\theta(s) - K\theta_1(s) = 0$$

$$\theta(s)\left[s^2 J_2 + s(B_{12}+B) + K\right] - \theta_1(s)\left[sB_{12} + K\right] = 0$$

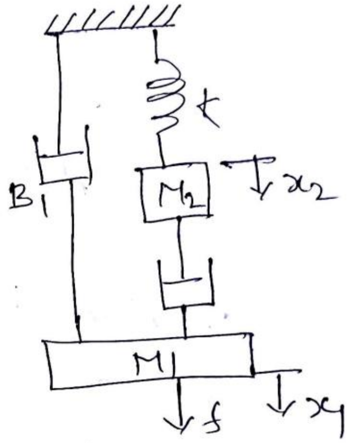
$$\theta_1(s) = \frac{s^2 J_2 + s(B_{12}+B) + K}{(sB_{12}+K)}\,\theta(s)$$

Substitute $\theta_1(s)$ in ①

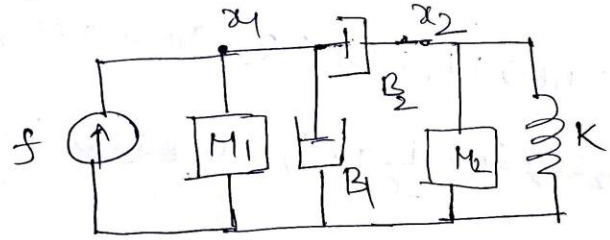
$$\left[J_1 s^2 + sB_{12} + K\right]\frac{\left[J_2 s^2 + s(B_{12}+B) + K\right]\theta(s)}{sB_{12}+K} - (sB_{12}+K)\theta(s) = T(s)$$

$$\frac{\theta(s)}{T(s)} = \frac{(sB_{12}+K)}{(J_1 s^2 + sB_{12} + K)\left[J_2 s^2 + s(B_{12}+B) + K\right] - (sB_{12}+K)^2}$$

→ Draw the equivalent mechanical system of the given system and hence write the set of equilibrium equations for it and obtain electrical analogous circuits using
(i) F-V analogy
(ii) F-I analogy



Sol: Equivalent Mechanical System:



There are two displacements and hence two nodes x_1 and x_2

At node 1:

$$M_1\frac{d^2x_1}{dt^2} + B_1\frac{dx_1}{dt} + B_2\frac{d(x_1-x_2)}{dt} = F$$

$$M_1s^2X_1 + B_1sX_1 + B_2s(X_1-X_2) = F \longrightarrow (1)$$

At Node 2

$$M_2\frac{d^2x_2}{dt^2} + Kx_2 + B_2\frac{d}{dt}(x_2-x_1) = 0$$

$$M_2s^2X_2 + KX_2 + B_2s(X_2-X_1) = 0 \longrightarrow (2)$$

Illustration:

(i) F-V analogy:

$F \to V, \; M \to L, \; B \to R, \; K \to \frac{1}{C}, \; x \to q$

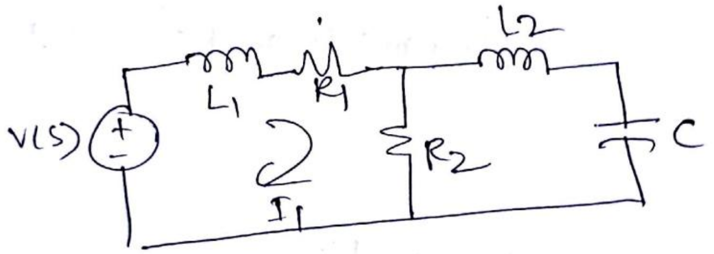
(1) $\Rightarrow$ $V = L_1 s^2 q_1(s) + R_1 s q_1(s) + R_2 s\left[q_1(s) - q_2(s)\right] \to$ (i)

(2) $\Rightarrow$ $0 = L_2 s^2 q_2(s) + \frac{1}{C} q_2(s) + R_2 s\left[q_2(s) - q_1(s)\right]$

But $q(s) = \frac{I(s)}{s}$, $I(s) = s\,q(s)$

$\therefore V(s) = L_1 s I_1(s) + R_1 I_1(s) + R_2\left[I_1(s) - I_2(s)\right] \to$ for loop (1)

$0 = L_2 s I_2(s) + \frac{1}{sC} I_2(s) + R_2\left[I_2(s) - I_1(s)\right]$ $\to$ for loop (2)

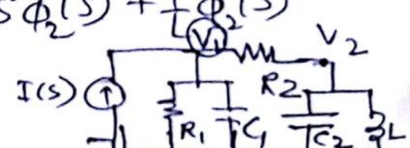


F-I analogy: $F \to I, \; M \to C, \; B \to \frac{1}{R}, \; K \to \frac{1}{L}, \; x \to \phi$

$$\phi(s) = \frac{V(s)}{s}$$

$\therefore I(s) = C_1 s^2 \phi_1(s) + \frac{1}{R_1} s\,\phi_1(s) + \frac{1}{R_2} s\left[\phi_1(s) - \phi_2(s)\right]$

$0 = \frac{1}{R_2} s\left[\phi_2(s) - \phi_1(s)\right] + C_2 s^2 \phi_2(s) + \frac{1}{L}\phi_2(s)$

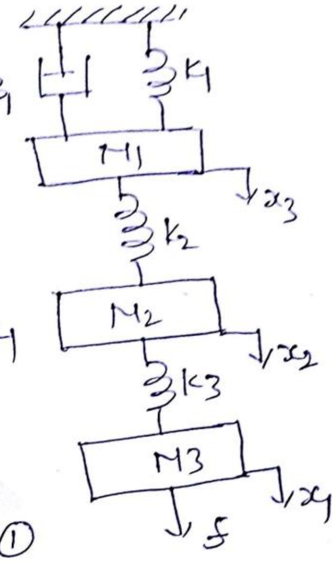


But $s\phi(s) = V(s)$

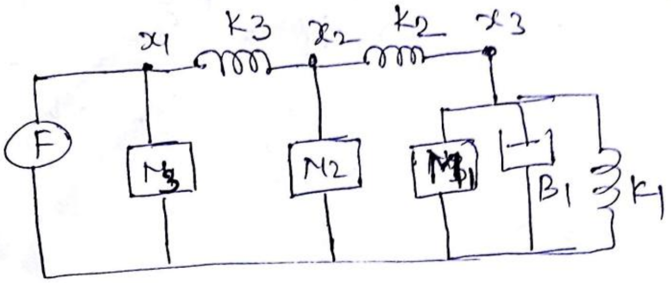
$\therefore I(s) = C_1 s V_1(s) + \frac{V_1(s)}{R_1} + \frac{1}{R_2}\left[V_1(s) - V_2(s)\right]$

$0 = \frac{1}{R_2}\left[V_2(s) - V_1(s)\right] + C_2 s V_2(s) + \frac{1}{sL_2} V_2(s)$

→ Draw the equivalent mechanical system and obtain its electrical analogous circuits from the set of equilibrium equations.



Sol: Equivalent Mechnical System



At Node 'x_1':

$$F = M_3 \frac{d^2 x_1}{dt^2} + K_3(x_1 - x_2) \longrightarrow (1)$$

At Node 'x_2':

$$0 = M_2 \frac{d^2 x_2}{dt^2} + K_3(x_2 - x_1) + K_2(x_2 - x_3) \longrightarrow (2)$$

At Node 'x_3':

$$0 = M_1 \frac{d^2 x_3}{dt} + B_1 \frac{dx_3}{dt} + K_1 x_3 + K_2(x_3 - x_2) \longrightarrow (3)$$

Apply LT to equations (1), (2) and (3)

$$(1) \Rightarrow \quad F = M_3 s^2 X_1 + K_3(X_1 - X_2) \longrightarrow (4)$$

$$(2) \Rightarrow \quad 0 = M_2 s^2 X_2 + K_3(X_2 - X_1) + K_2(X_2 - X_3) \longrightarrow (5)$$

$$(3) \Rightarrow \quad 0 = M_1 s^2 X_3 + B_1 s X_3 + K_1 X_3 + K_2(X_3 - X_2) \longrightarrow (6)$$

(i) F - V analogy:

$F \to V$, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $x \to q$

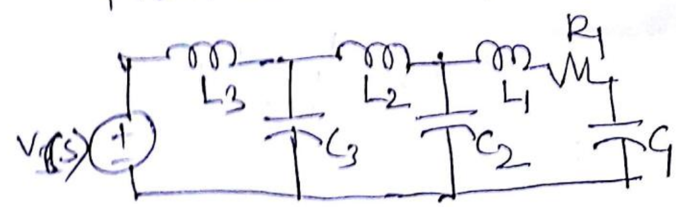
$\therefore$ (4) $\Rightarrow$

$$V = L_3 s^2 q_1(s) + \frac{1}{C_3}[q_1(s) - q_2(s)]$$

$$(5) \Rightarrow \quad 0 = L_2 s^2 q_2(s) + \frac{1}{C_3}[q_2(s) - q_1(s)] + \frac{1}{C_2}[q_2(s) - q_3(s)]$$

$$(6) \Rightarrow \quad 0 = L_1 s^2 q_3(s) + R_1 s q_3(s) + \frac{1}{C_1} q_1(s) + \frac{1}{C_2}[q_3(s) - q_2(s)]$$

From the above equations



F - I analogy: $F \to I$, $M \to C$, $B \to \frac{1}{R}$, $K \to 1/L$

$$(4) \Rightarrow I(s) = C_3 s^2 \phi_1(s) + \frac{1}{L_3}[\phi_1(s) - \phi_2(s)]$$

$$0 = C_2 s^2 \phi_2(s) + \frac{1}{L_3}[\phi_2(s) - \phi_1(s)] + \frac{1}{L_2}[\phi_2(s) - \phi_3(s)]$$

$$0 = C_1 s^2 \phi_3(s) + \frac{1}{R_1} s \phi_3(s) + \frac{1}{L_1}\phi_3(s) + \frac{1}{L_2}[\phi_3(s) - \phi_2(s)]$$